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TWO-FLUID MODEL OF TWO-PHASE FLOW IN A PIN BUNDLE OF A NUCLEAR REACTOR

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Abstract—By considering two-phase flow as a field which is subdivided into two turbulent single-phase regions with moving boundaries separating the two constituent phases, such that the differential balances for three-dimensional turbulent flow hold for each subregion and for the interface, we perform the Eulerian area averaging over the cross-sectional area of each phase in a given channel and segment averaging of transverse momentum equation along the phase intercepts at the interchannel boundaries. To simplify the governing equations obtained as a result of these operations, we invoke the assumption that the motion of the fluid in each phase is dominantly in axial direction, that is the transverse components of velocity are small compared to axial components.

We further assume that the variation of axial component of velocity within a channel is much stronger than the variation along the axial direction. We also assume that similar arguments can also be applied to the variation of enthalpy in a channel. As a result of these considerations, we obtain two sets of continuity, momentum, and energy equations describing motion of each phase in the axial direction. The phasic interaction terms which appear in these equations are governed by interfacial transfer conditions obtained from interface balances. The segment-averaged transverse-momentum equation for each phase provides the governing equation for cross flow.

NOMENCLATURE

a,	cross-sectional area normal to z axis;
a_k,	cross-sectional area for kth phase;
E,	energy at interface defined by equation (6);
$\tilde{e}$,	$\tilde{u} + \frac{1}{2}(\tilde{v}')^2$, sum of mass weighted internal energy and turbulent kinetic energy;
g,	acceleration due to gravity;
H_{21},	local curvature ($H_{21} > 0$ if phase 2 is the dispersed phase);
$\tilde{h}$,	$\tilde{e} + \bar{P}/\bar{\rho}$;
I,	unit tensor;
$\tilde{i}$,	enthalpy;
K,	unit vector in z direction;
$\bar{P}$,	time-averaged pressure;
$\bar{q}$,	time averaged heat flux;
s_k,	total intercept of kth phase at the interchannel gap;
s_I,	total perimeter of the interphase at a given section;
t,	time;
$\tilde{u}$,	specific internal energy;
$\tilde{v}$,	mass weighted fluid velocity vector;
W_{kij},	cross flow for the kth phase per unit axial length between channels i and j;
(x, y, z),	coordinate system at the center of a channel;
(x', y', z'),	coordinate system at the interchannel gap.

Greek symbols

Π_k,	$-\bar{P}_k I + \bar{\tau}_k$;
$\bar{\rho}$,	time-averaged fluid density;
$\bar{\tau}$,	time-averaged stess tensor.

Subscripts

e,	boundary between interconnected channels;
I,	vapor–liquid interface;
i,	channel i;
k,	kth phase;
n,	normal direction;
t,	tangential direction;
x,	directed along x' direction;
z,	directed along z or z' direction.

Superscripts

T,	turbulent;
t,	total;
$'$,	fluctuating quantities about mass averaged variables;
$''$,	fluctuating quantities about time averaged variables;
$\bar{}$,	time averaged quantity;
$\tilde{}$,	mass averaged quantity.

INTRODUCTION

AN ACCURATE prediction of both single and two-phase thermal hydraulics of a pin bundle for thermal or fast reactors is of extreme importance both to the design and the safety of these reactors. The fluid flow and heat transfer in a pin bundle is extremely complex, consequently with the exception of the formulation by Chawla and Ishii [1], all previous attempts (see for example [2–5]) at formulating the governing equations both for single and two-phase flows have utilized heuristic macroscopic balances using finite control volumes (e.g. subchannels) for mass, momentum, and energy.

It has long been recognized that the cross-sectional area and volume averaging are very useful tools in formulating governing equations for fluid flow and heat transfer in very complex geometries. The volume averaging has been used very successfully in treating multicomponent flows in porous media [6–8]. As a result of area averaging, the three-dimensional field equations are reduced to quasi-one-dimensional forms. The transfer of the momentum and energy between the fluid and wall is expressed by empirical correlations or by simplified models to supplement the information on changes of variables in direction normal to the main flow direction which are lost within a subchannel as a result of area averaging. However, the application of area averaging alone in a pin-bundle geometry does not yield a complete description of the momentum exchange between the channels at the interchannel boundaries. This consideration in turn has led to the use of segment averaging along the interchannel boundary of momentum equation in transverse (to the gap between the pins) direction to provide an equation for cross flow. These formal procedures were utilized for the first time by Chawla and Ishii [1] in the formulation of the governing equation for a drift flux model of two-phase flow in a pin-bundle geometry. The segment and area averaging has been proposed originally by Vernier and Delhaye [9] and Kocamustafaogullari [10] for laminar flows. However, these authors have not formally applied their methodology to a pin bundle configuration or extended to turbulent flows.

The drift flux model (or mixture model) is formulated by considering the mixture as a whole, rather than two-phases separately. The drift flux model thus requires only four field equations namely, continuity, momentum, and energy equations for the mixture, and the continuity equation for one of the phases, say vapor [1, 11]. On the other hand, a two-fluid model of a two-phase flow is formulated in terms of two sets of conservation equations governing the balance of mass, momentum, and energy of each phase [12]. Since the macroscopic fields of one phase are not independent of the other phase, the interaction terms which couple the transport of mass, momentum, and energy of each phase across the interphases appear in the field equation, whereas in the drift flux model which considers the mixture as whole, these interaction terms cancel each other. In the two-fluid model formulation, the transport processes of each phase are expressed by their own balance equations, therefore it is expected that the model can predict more detailed changes and phase interactions than the drift flux model. Although the drift flux model is simpler than the two-fluid model, it requires some drastic constitutive assumptions since it has only four field equations in contrast to six field equations in the two-fluid model. Therefore, it is natural that some of the characteristics of two-phase flow will be lost in the drift model. The drift flux model is generally useful in analyzing two-phase flows where there exists a strong coupling between the motion of the two phases, and the information desired is the response of total mixture and not that of each constituent phase separately. For example, in the dynamic analysis of two-phase flow systems where the response of total system is desired such as in the analysis of thermohydraulic flow instability problem in the boiling channels [13, 14]. Two-phase flow problems involving a sudden acceleration of one phase may not be appropriately described by the drift flux model. In these cases, inertia terms of each phase should be considered, that is, by use of the two-fluid model.

Previous studies have indicated that unless phasic interaction terms are accurately modeled in the two-fluid model, the numerical instabilities are frequently encountered in the numerical solution of these models [15–17]. Recent studies by Lahey *et al.* [18] and Lyczkowski *et al.* [19] have demonstrated that virtual mass originating from momentum interaction between the two phases has a considerable effect on improving numerical stability and efficiency. Another approach to achieving numerical stability is the inclusion of 'artificial viscosity' in the numerical algorithm to damp out high frequency oscillations occurring possibly due to imprecise modeling. This approach is currently being followed by Amsden and Harlow [20] in their two-fluid digital computer codes. In spite of these shortcomings of the two-fluid model, there is, however, no substitute available for modeling accurately two-phase phenomena where two phases are weakly coupled. The objective of the present paper is to obtain the governing equations for two-fluid model for two-phase flows in a pin bundle geometry.

BASIC EQUATIONS

We view two-phase flow as a field which is subdivided into two turbulent single-phase regions with moving boundaries separating the two constituent phases, such that the differential balances for turbulent flow hold for each subregion and for the interface, wherein the latter differential balances which accounts for singular characteristics of the interface, we further assume that all interfaces are identical, of zero thickness, and have the same interface velocity. With the exception of the interface velocity, we assume that all other singular transferrable properties of the interface are turbulent in nature. Since the turbulent fields in each subregion are unsteady because of the moving and deforming interfaces, one must view conceptually these turbulent balances as ensemble averages which are constructed with an assumption that all the samples of two-phase flows in an ensemble are statistically identical such that if all are observed at a given instant of time, a given point in each sample is surrounded by the same phase or is located at the interface implying the structure of two-phase flows and the geometry of interfaces are identical between the samples. By assuming further that the two-phase flow is temporally stationary, then by the ergodic hypothesis the ensemble averages at a given instant of time become equal to temporal averages. This methodology clearly implies extension of the continuum approach applicable to a single-phase unsteady turbulent flow. Consequently, the governing equations for each of the bulk phases are identical to the basic Reynolds' equation for nonsteady turbulent flow. This basic consideration is identical to that utilized by Slattery [21]. In view of the assumption that interfaces consisting of singular surfaces of zero thickness rather than assuming three-dimensional regions of finite thicknesses, the resulting jump conditions are identical to those that will be obtained by volume integration of single-phase turbulent flow over material volume containing a phase interface in the manner as done by Slattery [21].

The previously discussed methodology for obtaining basic governing equations differs both from the procedure by Ishii [22] and that by Delhaye [23]. Ishii directly formulates a two-fluid model for three-dimensional two-phase flows by time averaging the two-phase mixture rather than each component as we have carried out. On the other hand, Delhaye recommends double time averaging of two-phase mixture to smooth out the discontinuities in the time derivatives of single time-averaged quantities in the field.

Bulk phase

In view of the above discussions, the basic governing equations for the bulk phase can be written as (see [24] for detailed derivation):

continuity

$$\frac{\partial \bar{\rho}_k}{\partial t} + \nabla \cdot (\bar{\rho}_k \tilde{v}_k) = 0; \tag{1}$$

momentum

$$\frac{\partial \bar{\rho}_k \tilde{v}_k}{\partial t} + \nabla \cdot (\bar{\rho}_k \tilde{v}_k \tilde{v}_k) = -\nabla \bar{P}_k + \nabla \cdot (\bar{\tau}_k + \tau_k^T) + \bar{\rho}_k g_k; \tag{2}$$

energy

$$\frac{\partial \bar{\rho}_k \tilde{h}_k}{\partial t} + \nabla \cdot (\bar{\rho}_k \tilde{h}_k \tilde{v}_k) = -\nabla \cdot (\bar{q}_k + q_k^T) + \frac{\partial \bar{P}_k}{\partial t} + \tilde{v}_k \cdot \nabla \bar{P}_k + (\bar{\tau}_k + \tau_k^T) \colon \nabla \tilde{v}_k. \tag{3}$$

In the above equations, we have utilized a mass-weighted-averaging procedure rather than conventional time averaging. The former procedure is well known in the studies of gas mixtures. The mass-weighted averaging greatly simplifies the governing equations. For example, it eliminates terms like $\overline{\rho_k'' v_k''}$, $\overline{\tilde{v}_k \rho_k'' v_k''}$, and $\overline{\rho_k'' v_k'' v_k''}$ in the continuity and momentum equations. Analogous simplifications are also achieved in the energy equations. In fact, as a result of a mass weighting, the form of the equations is the same as for incompressible flow. To illustrate this, let us for example consider the time average of convective flux of a quantity

$$\begin{aligned}\overline{\rho_k \Psi_k v_k} &= \overline{(\bar{\rho}_k + \rho_k'')(\tilde{\Psi}_k + \Psi_k')(\tilde{v}_k + v_k')} \\ &= \bar{\rho}_k \tilde{\Psi}_k \tilde{v}_k + \overline{\rho_u'' \tilde{\Psi}_k \tilde{v}_k} + \overline{\rho_k \tilde{\Psi}_k v_k'} \\ &\quad + \overline{\rho_k \Psi_k' \tilde{v}_k} + \overline{\rho_k \Psi_k' v_k'}.\end{aligned}$$

In view of definition of mass-weighted quantity, i.e.,

$$\tilde{\Psi}_k = \frac{\overline{\rho_k \Psi_k}}{\bar{\rho}_k}, \quad \tilde{v}_k = \frac{\overline{\rho_k v_k}}{\bar{\rho}_k}$$

we have

$$\overline{\tilde{\Psi}_k \tilde{v}_k \rho_k''} = \overline{\rho_k \tilde{\Psi}_k v_k'} = \overline{\rho_k \Psi_k' \tilde{v}_k} = 0$$

which results in

$$\overline{\rho_k \Psi_k v_k} = \bar{\rho}_k \tilde{\Psi}_k \tilde{v}_k + \overline{\rho_k \Psi_k' v_k'}.$$

The form of the above equation is clearly the same as it would be for incompressible flow.

Interface

A detailed statement of underlying simplifying assumptions and the derivation of interface balance equations is given in [24]. These balance equations are reproduced here for ready reference:

mass

$$\sum_{k=1}^{2} \{\bar{\rho}_k n_k \cdot (\tilde{v}_k - v_I)\} = \sum_{k=1}^{2} \dot{m}_k = 0; \tag{4}$$

momentum

$$\begin{aligned}&\sum_{k=1}^{2} \{\dot{m}_k \tilde{v}_k - n_k \cdot (\bar{\Pi}_k + \tau_k^T)\} \\ &= \sum_{k=1}^{2} \left[\left(\frac{\dot{m}_k^2}{\bar{\rho}_k} + \bar{P}_k + \tau_{knn}^t\right) n_k + \tau_{knt}^t\right] \\ &= \nabla_s \cdot n\sigma n;\end{aligned} \tag{5}$$

energy

$$\sum_{k=1}^{2} \{\dot{m}_k(\tilde{e}_k + \tfrac{1}{2}\tilde{v}_k^2) + n_k \cdot (\bar{q}_k + q_k^T) - n_k \cdot [(\bar{\Pi}_k + \tau_k^T) \cdot \tilde{v}_k]\} = \sum_k E_k = 0 \tag{6}$$

where

$$\tau_k^t = \bar{\tau}_k + \tau_k^T.$$

AREA AVERAGED EQUATIONS

We assume that both in the liquid and in the vapor phase, the motion of the fluids is dominantly in the axial direction, that is the transverse components of velocity are small compared to the axial component. We further assume that within a channel the variation of the axial component v_{k_z} is much stronger than the variation along the axial direction. This behavior is analogous to that which exists in a boundary layer so that the longitudinal or axial length scale L (in which the axial variations in v_{k_z} take place) is very much larger than the length scale δ in a transverse direction (over which transverse variations in v_{kz} take place) i.e. $\delta/L \ll 1$. We further assume that similar arguments can be applied to the variation of enthalpy or temperature in a channel. In conclusion, it is assumed that the boundary layer approximation can be applied.

Continuity equation

The application of Leibnitz theorem (A.2) and divergence theorem (A.6) (see Appendix) to equation (1) yields for a channel denoted by i in the subassembly (see Fig. 1):

$$\frac{\partial}{\partial t}(a_k\langle\langle\bar{\rho}_k\rangle\rangle)_i + \frac{\partial}{\partial z}(a_k\langle\langle\bar{\rho}_k\tilde{v}_{kz}\rangle\rangle)_i = -\left[\int_{\mathscr{C}_I}\bar{\rho}_k(\tilde{v}_k - v_I)\cdot n_k\frac{\mathrm{d}\mathscr{C}}{n_k\cdot n_{kI}}\right]_i - \sum_{j=1}^{N_i}\left(\int_{\mathscr{C}_{ke}}\bar{\rho}_k\tilde{v}_k\cdot n_k\frac{\mathrm{d}\mathscr{C}}{n_k\cdot n_{ke}}\right)_{ij}. \quad (7)$$

If we let

$$\frac{1}{s_I}\int_{\mathscr{C}_I}F_k\frac{\mathrm{d}\mathscr{C}}{n_k\cdot n_{kI}} = \langle F_k\rangle \quad (8a)$$

$$\left(\int_{\mathscr{C}_{ke}}\bar{\rho}_k\tilde{v}_k\cdot n_k\frac{\mathrm{d}\mathscr{C}}{n_k\cdot n_{ke}}\right)_{ij} = W_{kij}. \quad (8b)$$

Equation (7) becomes

$$\frac{\partial}{\partial t}(a_k\langle\langle\bar{\rho}_k\rangle\rangle)_i + \frac{\partial}{\partial z}(a_k\langle\langle\bar{\rho}_k\tilde{v}_{kz}\rangle\rangle)_i = -(s_I\langle\dot{m}_k\rangle)_i - \sum_j W_{kij}. \quad (9)$$

Here ($\langle\langle\ldots\rangle\rangle$) defines the area averaged value of a quantity, i.e.

$$\langle\langle F_k\rangle\rangle(z,t) = \frac{1}{a_k}\iint_{a_k}F_k(x,y,z,t)\mathrm{d}a. \quad (10)$$

In a typical reactor application, the point-wise variation of the density in a phase at a given axial location in a channel is generally small as compared to the axial variation. It is therefore assumed that

$$\langle\langle\rho_k\rangle\rangle(z,t) \simeq \bar{\rho}_k(z,t). \quad (11)$$

In view of assumption (11), equation (9) becomes

$$\frac{\partial}{\partial t}(a_k\bar{\rho}_k)_i + \frac{\partial}{\partial z}(a_k\bar{\rho}_k\langle\langle\tilde{v}_{kz}\rangle\rangle)_i = -(s_I\langle\dot{m}_k\rangle)_i - \sum_j W_{kij}. \quad (12)$$

In view of interface balance equation (4) and definition (8a) we obtain the following interfacial area averaged mass-transfer condition

$$\sum_{k=1}^{2}\langle\dot{m}_k\rangle = 0. \quad (13)$$

Momentum equation

The application of equations (A.2) and (A.6) to equation (2) yields

$$\frac{\partial}{\partial t}(a_k\langle\langle\bar{\rho}_k\rangle\rangle\{\tilde{v}_k\})_i + \frac{\partial}{\partial z}(a_k\langle\langle\bar{\rho}_k\tilde{v}_{kz}\tilde{v}_k\rangle\rangle)_i = -\left[\int_{\mathscr{C}_I}\bar{\rho}_k(\tilde{v}_k - v_I)\cdot n_k\tilde{v}_k\frac{\mathrm{d}\mathscr{C}}{n_k\cdot n_{kI}}\right]_i - \sum_j\left[\int_{\mathscr{C}_{ke}}\bar{\rho}_k\tilde{v}_k\cdot n_k\tilde{v}_k\frac{\mathrm{d}\mathscr{C}}{n_k\cdot n_{ke}}\right]_{ij} + \frac{\partial}{\partial z}[a_k\langle\langle(\bar{\Pi}_{kz} + \tau_{kz}^T)\rangle\rangle]_i + \sum_j\left[\int_{\mathscr{C}_{ke}}(\bar{\Pi}_k + \tau_k^T)\cdot n_k\frac{\mathrm{d}\mathscr{C}}{n_k\cdot n_{ke}}\right]_{ij} + \left[\int_{\mathscr{C}_t}(\bar{\Pi}_k + \tau_k^T)\cdot n_k\frac{\mathrm{d}\mathscr{C}}{n_k\cdot n_{kI}}\right]_i + (\langle\langle\bar{\rho}_k\rangle\rangle a_k)_i g_k \quad (14)$$

where $\mathscr{C}_t = \mathscr{C}_{kw} + \mathscr{C}_I$, subscript w stands for a wall or an impenetrable surface, and ($\{\ldots\}$) denotes a mass weighted, area averaged quantity, i.e.

$$\{F_k\} = \frac{\langle\langle\bar{\rho}_kF_k\rangle\rangle}{\langle\langle\bar{\rho}_k\rangle\rangle}. \quad (15)$$

From the use of equation (11), the above relationship becomes

$$\{F_k\} \simeq \langle\langle F_k\rangle\rangle. \quad (16)$$

In view of the assumed applicability of boundary-layer approximation to the flow in the pin bundle for each phase the normal components of the vector momentum equation (14) are small compared to the axial component, furthermore,

$$\langle\langle\bar{P}_k\rangle\rangle \simeq \bar{P}_k(z,t). \quad (17)$$

The above assumption is consistent with the boundary-layer approximation applicable to vertical flow in a pin bundle. In view of these simplifications, the momentum equation (14) simplifies to

$$\frac{\partial}{\partial t}(\bar{\rho}_ka_k\langle\langle\tilde{v}_{kz}\rangle\rangle)_i + \frac{\partial}{\partial z}(a_k\bar{\rho}_k\langle\langle\tilde{v}_{kz}^2\rangle\rangle)_i = -\frac{\partial}{\partial z}(a_k\langle\langle\bar{P}_k\rangle\rangle)_i + \frac{\partial}{\partial z}(a_k\langle\langle\tau_{kzz} + \tau_{kzz}^T\rangle\rangle)_i - \left[\int_{\mathscr{C}_t}\bar{P}_kK\cdot n_k\frac{\mathrm{d}\mathscr{C}}{n_k\cdot n_{kI}}\right]_i + \left[\int_{\mathscr{C}_I}(\bar{\tau}_k\cdot K + \tau_k^T\cdot K)\cdot n_k\frac{\mathrm{d}\mathscr{C}}{n_k\cdot n_{kI}}\right]_i + \left[\int_{\mathscr{C}_{kw}}(\bar{\tau}_k\cdot K + \tau_k^T\cdot K)\cdot n_k\frac{\mathrm{d}\mathscr{C}}{n_k\cdot n_{kw}}\right]_i - \left[\int_{\mathscr{C}_{ke}}\bar{P}_kK\cdot n_k\frac{\mathrm{d}\mathscr{C}}{n_k\cdot n_{ke}}\right]_i$$

$$+\left[\int_{\mathscr{C}_{ke}} (\bar{\tau}_{kz} + \tau_{kz}^T)\cdot n_k \frac{d\mathscr{C}}{n_k \cdot n_{ke}}\right]_{ij}$$

$$-\left[\int_{\mathscr{C}_I} \bar{\rho}_k(\tilde{v}_k - v_I)\cdot n_k \tilde{v}_k \cdot K \frac{d\mathscr{C}}{n_k \cdot n_{kI}}\right]_i$$

$$-\sum_j\left[\int_{\mathscr{C}_{ke}} \bar{\rho}_k \tilde{v}_k \cdot n_k \tilde{v}_{kz} \frac{d\mathscr{C}}{n_k \cdot n_{ke}}\right]_{ij} - (\bar{\rho}_k a_k)_i g_z \quad (18)$$

where we have assumed that the gravity is the only body force. The above equations can be simplified further by noting that

$$\int_{\mathscr{C}_{ke}} K \cdot n_k \frac{d\mathscr{C}}{n_k \cdot n_{ke}} = 0, \quad (19)$$

$$\sum_{l=I,w}\int_{\mathscr{C}_l} K \cdot n_k \frac{d\mathscr{C}}{n_k \cdot n_{kl}} = -\frac{\partial a_k}{\partial z}, \quad (20)$$

$$-s_{kw}\tau_{kw} = \int_{\mathscr{C}_{kw}} (n_k \cdot \bar{\tau}_k)\cdot K \frac{d\mathscr{C}}{n_k \cdot n_{kw}}, \quad \tau_k^T|_{\text{wall}} = 0. \quad (21)$$

Equation (18) becomes

$$\frac{\partial}{\partial t}(a_k\bar{\rho}_k \langle\!\langle \tilde{v}_{kz}\rangle\!\rangle)_i + \frac{\partial}{\partial z}(a_k\bar{\rho}_k \langle\!\langle \tilde{v}_{kz}^2\rangle\!\rangle)_i$$

$$= -\frac{\partial}{\partial z}(a_k\langle\!\langle \bar{P}_k\rangle\!\rangle)_i + \frac{\partial}{\partial z}(a_k\langle\!\langle \tau_{kzz}^t\rangle\!\rangle)_i - (s_{kw}\tau_{kw})_i$$

$$+\sum_j\left[\int_{\mathscr{C}_{ke}} (n_k\cdot\tau_k^t - \bar{\rho}_k\tilde{v}_k\tilde{v}_k\cdot n_k)\cdot K \frac{d\mathscr{C}}{n_k\cdot n_{ke}}\right]_{ij}$$

$$+\left[\int_{\mathscr{C}_I} (-\dot{m}_k\tilde{v}_k - \bar{P}_k n_k + n_k\cdot\tau_k^t)\cdot K\frac{d\mathscr{C}}{n_k\cdot n_{kI}}\right]_i$$

$$-\left[\int_{\mathscr{C}_{kw}} \bar{P}_k n_k \cdot K \frac{d\mathscr{C}}{n_k\cdot n_{kw}}\right]_i - (a_k\bar{\rho}_k g_z)_i. \quad (22)$$

In equation (22), various terms retained are not necessarily of the same order, for example the second term on the RHS is of lower than the third term on the RHS, however, the former term represents the momentum diffusion in the axial direction and, therefore, is expected to contribute to numerical stability and thus, it is advantageous to maintain the presence of this term in the equation.

To obtain the interface balance condition, we let

$$M_k = -\dot{m}_k\tilde{v}_k - n_k\bar{P}_k + \tau_k^t\cdot n_k$$

$$= M_k^\Gamma + M_k^n n_k + M_k^t - \langle\bar{P}_k\rangle n_k + \langle\tau_k^t\rangle\cdot n_k \quad (23)$$

where

$$M_k^\Gamma = -\dot{m}_k\tilde{v}_k, \quad M_k^n = (\langle\bar{P}_k\rangle - \bar{P}_k) - (\langle\tau_{knn}^t\rangle - \tau_{knn}^t), \quad (24a)$$

$$M_k^t = -(\langle\tau_{knt}^t\rangle - \tau_{knt}^t), \quad M_k^d = M_k^n n_k + M_k^t, \quad (24b)$$

$$n_k\cdot\tau_k^t = (\bar{\tau}_k + \tau_k^T)\cdot n_k$$

$$= (\bar{\tau}_{knn} + \tau_{knn}^T)n_k + (\bar{\tau}_{knt} + \tau_{knt}^T). \quad (25)$$

The individual components of total momentum transfer M_k at the interface have the following physical meaning: M_k^Γ denotes the local momentum transfer due to mass transfer at the interface, M_k^n denotes the local form or pressure drag, M_k^t the local shear stress. In view of equation (5), we can now write the local interface balance for momentum as

$$\sum_k M_k = \nabla_s\cdot n\sigma n. \quad (26)$$

By noting that $v_{It} = \tilde{v}_{kt}$, and

$$M_k^\Gamma = -\dot{m}_k\tilde{v}_k = -\dot{m}_k(\tilde{v}_k - v_I) - \dot{m}_k v_I$$

$$= -\dot{m}_k(\tilde{v}_k - v_I)\cdot n_k n_k - \dot{m}_k v_I$$

$$= -\frac{\dot{m}_k^2}{\bar{\rho}_k}n_k - \dot{m}_k v_I, \quad (27a)$$

$$\sum_k M_k^\Gamma = -\sum_k \dot{m}_k\tilde{v}_k = -\sum_k\frac{\dot{m}_k^2}{\bar{\rho}_k}n_k \quad (27b)$$

$$-\sum_k \dot{m}_k v_I = 0, \quad (27c)$$

It follows from equations (26), (23), (24) and (25) that

$$\sum_k\left(-\frac{\dot{m}_k^2}{\bar{\rho}_k} - \bar{P}_k + \tau_{knn}^t\right)n_k = 2H_{21}\sigma n_1 \quad (28a)$$

or,

$$\left[-\dot{m}_1^2\left(\frac{1}{\bar{\rho}_1} - \frac{1}{\bar{\rho}_2}\right) - (\bar{P}_1 - \bar{P}_2) + (\tau_{1nn}^t - \tau_{2nn}^t)\right] = 2H_{21}\sigma \quad (28b)$$

and

$$\sum_k (M_k^t + \langle\tau_{knt}^t\rangle) = 0. \quad (28c)$$

Equation (28b) yields

$$-\langle\dot{m}_1^2\rangle\left(\frac{1}{\bar{\rho}_1} - \frac{1}{\bar{\rho}_2}\right) - (\langle\bar{P}_1\rangle - \langle\bar{P}_2\rangle)$$

$$+ (\langle\tau_{1nn}^t\rangle - \langle\tau_{2nn}^t\rangle) = 2\langle H_{21}\rangle\sigma. \quad (29)$$

When the above equation is supplemented with the equation of state, one can then determine interface pressures $\langle\bar{P}_1\rangle$ and $\langle\bar{P}_2\rangle$.

The combination of equations (29) and (28) yields

$$-(\langle\dot{m}_1^2\rangle - \dot{m}_1^2)\left(\frac{1}{\bar{\rho}_1} - \frac{1}{\bar{\rho}_2}\right) - (\langle\bar{P}_1\rangle - \bar{P}_1)$$

$$+ (\langle\bar{P}_2\rangle - \bar{P}_2) + (\langle\tau_{1nn}^t\rangle - \tau_{1nn}^t)$$

$$- (\langle\tau_{2nn}^t\rangle - \tau_{2nn}^t) = 2(\langle H_{21}\rangle - H_{21})\sigma. \quad (30)$$

If we assume that

$$\langle\dot{m}_k^2\rangle \simeq \dot{m}_k^2, \quad \langle H_{21}\rangle \simeq H_{21} \quad (31)$$

we obtain from equation (30)

$$\sum_k M_k^n n_k = 0. \quad (32)$$

From equation (27c) we obtain

$$K\cdot\left[\sum_k\int_{\mathscr{C}_I}\dot{m}_k v_I\frac{d\mathscr{C}}{n_1\cdot n_{1I}}\right]=s_I K\cdot\left[\sum_k \dot{m}_k\langle v_I\rangle\right]=0 \quad (33)$$

where we have assumed that $\dot{m}_k$ is nearly constant around the interface. Equations (24b) and (28c) imply that

$$\sum_k \tau^t_{knt}=0 \quad (34a)$$

from which it follows that

$$\sum_k\langle\tau^t_{knt}\rangle=0 \quad (34b)$$

or

$$\sum_k \tau_{kI}=0 \quad (34c)$$

where

$$-s_I\tau_{kI}=\int_{\mathscr{C}_I}\tau^t_{knt}\cdot K\frac{d\mathscr{C}}{n_k\cdot n_{kI}}. \quad (34d)$$

From equation (34) one can write

$$\sum_k \tau^t_{knt}-\sum_k\langle\tau^t_{knt}\rangle=0$$

or

$$\sum_k M^t_k=0. \quad (35)$$

The combination of equations (32) and (35) yields

$$\sum_k(M^t_k+M^n_k n_k)=\sum_k M^d_k=0 \quad (36)$$

from which we obtain

$$\sum_k\int_{\mathscr{C}_I}(M^t_k+M^n_k n_k)\cdot K\frac{d\mathscr{C}}{n_1\cdot n_{1I}}=K\cdot\sum_k\int_{\mathscr{C}_I}M^d_k\frac{d\mathscr{C}}{n_1\cdot n_{1I}}$$

or

$$\sum_k(\langle M^t_k\rangle+\langle M^n_k n_k\rangle)\cdot K=K\cdot\sum_k\langle M^d_k\rangle=0. \quad (37)$$

In view of equations (23) and (27a) we can write

$$K\cdot\int_{\mathscr{C}_I}M_k\frac{d\mathscr{C}}{n_k\cdot n_{kI}}=K\cdot\int[(M^\Gamma_k-\langle\bar{P}_k\rangle n_k+\langle\tau^t_k\rangle\cdot n_k)+(M^n_k n_k+M^t_k)]\frac{d\mathscr{C}}{n_k\cdot n_{kI}} \quad (38a)$$

or

$$s_I K\cdot\langle M_k\rangle=K\cdot\left[\int_{\mathscr{C}_I}\left(-\frac{\dot{m}_k^2}{\bar{\rho}_k}-\langle\bar{P}_k\rangle+\langle\tau^t_{knn}\rangle\right)n_k\frac{d\mathscr{C}}{n_k\cdot n_{kI}}\right]+\int_{\mathscr{C}_I}\langle\tau^t_{knt}\rangle\cdot K\frac{d\mathscr{C}}{n_k\cdot n_{kI}}-K\cdot\int_{\mathscr{C}_I}\dot{m}_k v_I\frac{d\mathscr{C}}{n_k\cdot n_{kI}}+K\cdot\langle M^d_k\rangle. \quad (38b)$$

The use of equations (20), (31) and (34d) yields

$$s_I K\cdot\langle M_k\rangle=\left(-\frac{\dot{m}_k^2}{\bar{\rho}_k}-\langle\bar{P}_k\rangle+\langle\tau^t_{knn}\rangle\right)\frac{\partial a_k}{\partial z}-s_I[\tau_{kI}+K\cdot(\dot{m}_k\langle v_I\rangle)\cdot K\cdot\langle M^d_k\rangle]. \quad (39)$$

We may also note that

$$\sum a_k=a$$

so that for constant cross sectional area a we have

$$\sum_k\frac{\partial a_k}{\partial z}=0. \quad (40)$$

The interface velocity in equation (33) can be related to fluid velocity and void fraction by considering

$$K\cdot\int_{\mathscr{C}_I}(\tilde{v}_k-v_I)\frac{d\mathscr{C}}{n_k\cdot n_{kI}}=K\cdot\int_{\mathscr{C}_I}(\tilde{v}_k-v_I)\cdot n_k n_k\frac{d\mathscr{C}}{n_k\cdot n_{kI}}$$

or

$$K\cdot(\langle\tilde{v}_k\rangle-\langle v_I\rangle)s_I=K\cdot\int_{\mathscr{C}_I}\frac{\dot{m}_k}{\bar{\rho}_k}n_k\frac{d\mathscr{C}}{n_k\cdot n_{kI}}=\frac{\dot{m}_k}{\bar{\rho}_k}\frac{\partial a_k}{\partial z} \quad (41a)$$

which yields

$$\langle v_I\rangle\cdot K=\langle\tilde{v}_{kz}\rangle-\frac{\dot{m}_k}{s_I\bar{\rho}_k}\frac{\partial a_k}{\partial z}. \quad (41b)$$

The use of expression (39) into equation (22) yields

$$\frac{\partial}{\partial t}(a_k\bar{\rho}_k\langle\langle\tilde{v}_{kz}\rangle\rangle)_i+\frac{\partial}{\partial z}(a_k\bar{\rho}_k\langle\langle\tilde{v}^2_{kz}\rangle\rangle)_i$$
$$=-\frac{\partial}{\partial z}(a_k\langle\langle\bar{P}_k\rangle\rangle)_i+\frac{\partial}{\partial z}(a_k\langle\langle\tau^t_{kzz}\rangle\rangle)_i-(s_{kw}\tau_{kw})_i+\sum_j\left[\int_{\mathscr{C}_{ke}}(n_k\cdot\tau^t_k-\bar{\rho}_k\tilde{v}_k\tilde{v}_k\cdot n_k)\cdot K\frac{d\mathscr{C}}{n_k\cdot n_{ke}}\right]_{ij}$$
$$-(a_k\rho_k g_z)_i+\left\{\left(-\frac{\dot{m}_k^2}{\bar{\rho}_k}-\langle\bar{P}_k\rangle+\langle\tau^t_{knn}\rangle\right)\frac{\partial a_k}{\partial z}-s_I[\tau_{kI}+(\dot{m}_k\langle v_I\rangle-\langle M^d_k\rangle)\cdot K]\right\}_i \quad (42)$$

where we have utilized equation (19) and the fact that total cross sectional area is constant. Momentum equation (41) is supplemented by interface balance equations (29), (33), (34), (37) and (40).

Energy equation

The application of equations (A.2) and (A.6) to enthalpy equation (3) gives

$$\frac{\partial}{\partial t}(a_k\bar{\rho}_k\langle\langle\tilde{h}_k\rangle\rangle)_i+\frac{\partial}{\partial z}(a_k\bar{\rho}_k\langle\langle\tilde{h}_k\tilde{v}_{kz}\rangle\rangle)_i$$
$$=-\frac{\partial}{\partial z}(a_k\langle\langle q^t_k\rangle\rangle)_i+\frac{\partial}{\partial t}(a_k\langle\langle\bar{P}_k\rangle\rangle)_i+\left[\langle\langle\tilde{v}_{kz}\rangle\rangle\frac{\partial}{\partial z}(a_k\langle\langle\bar{P}_k\rangle\rangle)\right]_i+\frac{\partial}{\partial z}(a_k\langle\langle K\cdot(\tau^t_k\cdot\tilde{v}_k)\rangle\rangle)_i$$
$$-\left[\langle\langle\tilde{v}_k\rangle\rangle\cdot\frac{\partial}{\partial z}(a_k\langle\langle\tau^t_k\cdot K\rangle\rangle)\right]_i$$

$$+ \{[\mathrm{COV}(\tilde{v}_k \cdot \nabla \bar{P}_k) - \mathrm{COV}(\tilde{v}_k \cdot \nabla \cdot \tau_k^t)]a_k\}_i$$

$$- \left\{ \int_{\mathscr{C}_{kw}} [\bar{q}_k - \langle\!\langle \tilde{v}_k \rangle\!\rangle \bar{P}_k + \langle\!\langle \tilde{v}_k \rangle\!\rangle \cdot \bar{\tau}_k] \cdot n_k \frac{\mathrm{d}\mathscr{C}}{n_k \cdot n_{kw}} \right\}_i$$

$$- \sum_j \left\{ \int_{\mathscr{C}_{ke}} [\bar{\rho}_k \tilde{h}_k \tilde{v}_k + q_k^t - \langle\!\langle \tilde{v}_k \rangle\!\rangle \bar{P}_k - (\tilde{v}_k \cdot \tau_k^t) + \langle\!\langle \tilde{v}_k \rangle\!\rangle \cdot \tau_k^t] \cdot n_k \frac{\mathrm{d}\mathscr{C}}{n_k \cdot n_{ke}} \right\}_{ij}$$

$$+ \left\{ \int_{\mathscr{C}_I} [- \dot{m}_k \tilde{h}_k - q_k^t \cdot n_k + (\langle\!\langle \tilde{v}_k \rangle\!\rangle - v_I) \cdot n_k \bar{P}_k + n_k \cdot (\tau_k^t \cdot \tilde{v}_k) - \langle\!\langle \tilde{v}_k \rangle\!\rangle \cdot (n_k \cdot \tau_k^t)] \frac{\mathrm{d}\mathscr{C}}{n_k \cdot n_{kI}} \right\}_i \quad (43)$$

where covariance is defined as

$$\mathrm{COV}(\psi_k \cdot \phi_k) = \langle \psi_k \cdot \phi_k \rangle - \langle \psi_k \rangle \cdot \langle \phi_k \rangle. \quad (44)$$

In view of equation (17) and the above definition, we have

$$\mathrm{COV}(\tilde{v}_k \cdot \nabla \bar{P}_k) = 0. \quad (45)$$

For negligible or linear distribution of shear in either phase (these assumptions are approximately valid in slug flow and annular two-phase flow, respectively) we have

$$\mathrm{COV}(\tilde{v}_k \cdot \nabla \cdot \tau_k^t) \simeq 0. \quad (46)$$

In reactor applications, the work done against shear stress (dissipation work) and work done by pressure are generally small compared to the heat flux term, therefore, the above assumption (46) is generally valid for almost all flow regimes. In view of boundary layer approximation

$$\langle\!\langle \tilde{v}_k \rangle\!\rangle \simeq \langle\!\langle \tilde{v}_{kz} \rangle\!\rangle K. \quad (47)$$

The use of equations (17), (19) and (47) yield

$$\langle\!\langle \tilde{v}_k \rangle\!\rangle \cdot \left[\int_{\mathscr{C}_{ke}} P_k n_k \frac{\mathrm{d}\mathscr{C}}{n_k \cdot n_{ke}} \right] \simeq 0. \quad (48)$$

Similarly

$$\langle\!\langle \tilde{v}_k \rangle\!\rangle \cdot \int_{\mathscr{C}_{kw}} \bar{P}_k n_k \frac{\mathrm{d}\mathscr{C}}{n_k \cdot n_{kw}} \simeq 0. \quad (49)$$

Further assuming that the dissipation work at the interchannel gap as compared to enthalpy convection and heat flux terms can also be neglected. This assumption will certainly be valid if velocity distribution in a phase is essentially flat, as in this case

$$\langle\!\langle \tilde{v}_k \rangle\!\rangle \simeq \tilde{v}_k \quad \text{and}$$

$$\int_{\mathscr{C}_{ke}} (\langle\!\langle \tilde{v}_k \rangle\!\rangle - \tilde{v}_k) \cdot \tau_k^t \cdot n_k \frac{\mathrm{d}\mathscr{C}}{n_k \cdot n_{ke}} \simeq 0. \quad (50)$$

In view of equations (47) and (21) we can write

$$\langle\!\langle \tilde{v}_{kz} \rangle\!\rangle \int_{\mathscr{C}_{kw}} K \cdot \bar{\tau}_k \cdot n_k \frac{\mathrm{d}\mathscr{C}}{n_k \cdot n_{kw}} = - \langle\!\langle \tilde{v}_{kz} \rangle\!\rangle s_{kw} \tau_{kw}. \quad (51)$$

Owing to boundary layer approximation, the fourth and fifth terms in equation (43) representing dissipation work are small compared to dissipation work done by the wall shear stress [see equation (51)]. In view of the above simplifications, equation (43) becomes

$$\frac{\partial}{\partial t}(a_k \bar{\rho}_k \langle\!\langle \tilde{h}_k \rangle\!\rangle)_i + \frac{\partial}{\partial z}(a_k \bar{\rho}_k \langle\!\langle \tilde{h}_k \tilde{v}_{kz} \rangle\!\rangle)_i$$

$$= - \frac{\partial}{\partial z}(a_k \langle\!\langle q_k^t \rangle\!\rangle)_i + \frac{\partial}{\partial t}(a_k \langle\!\langle \bar{P}_k \rangle\!\rangle)_i + \langle\!\langle \tilde{v}_{kz} \rangle\!\rangle \frac{\partial}{\partial z}(a_k \langle\!\langle \bar{P}_k \rangle\!\rangle)_i$$

$$- \left(\int_{\mathscr{C}_{kw}} \bar{q}_k \cdot n_k \frac{\mathrm{d}\mathscr{C}}{n_k \cdot n_{kw}} \right)_i + (\langle\!\langle \tilde{v}_{kz} \rangle\!\rangle s_{kw} \tau_{kw})_i - \sum_j \left[\int_{\mathscr{C}_k} (\bar{\rho}_k \tilde{h}_k \tilde{v}_k + q_k^t) \cdot n_k \frac{\mathrm{d}\mathscr{C}}{n_k \cdot n_{ke}} \right]_{ij}$$

$$+ \left\{ \int_{\mathscr{C}_I} [- \dot{m}_k \tilde{h}_k - q_k^t \cdot n_k + (\langle\!\langle \tilde{v}_k \rangle\!\rangle - v_I) \cdot n_k \bar{P}_k + n_k \cdot (\tau_k^t \cdot \tilde{v}_k) - \langle\!\langle \tilde{v}_k \rangle\!\rangle \cdot (n_k \cdot \tau_k^t)] \frac{\mathrm{d}\mathscr{C}}{n_k \cdot n_{kI}} \right\}_i. \quad (52)$$

To obtain an interphase balance condition for enthalpy we note that in pin bundle applications, the fluid speeds of interest are sufficiently low so that the terms like $(\langle\!\langle \tilde{v}_k \rangle\!\rangle - v_I) \cdot n_k \bar{P}_k$ and $n_k \cdot \tau_k^t \cdot (\tilde{v}_k - \langle\!\langle \tilde{v}_k \rangle\!\rangle$ at the interface can be neglected as compared to heat flux term $q_k^t \cdot n_k$. Thus,

$$\left\{ \int_{\mathscr{C}_I} [- \dot{m}_k \tilde{h}_k - q_k^t \cdot n_k + (\langle\!\langle \tilde{v}_k \rangle\!\rangle - v_I) \cdot n_k \bar{P}_k + n_k \cdot \tau_k^t \cdot (\tilde{v}_k - \langle\!\langle \tilde{v}_k \rangle\!\rangle)] \times \frac{\mathrm{d}\mathscr{C}}{n_k \cdot n_{kI}} \simeq \int_{\mathscr{C}_I} (- \dot{m}_k \tilde{h}_k - q_k^t \cdot n_k) \frac{\mathrm{d}\mathscr{C}}{n_k \cdot n_{kI}}. \right. \quad (53)$$

From the use of equations (6) and (23) we can write

$$\sum_k \left(E_k - \frac{\tilde{v}_k^2 \dot{m}_k}{2} - M_k \cdot \tilde{v}_k \right) = \sum_k \left(- \dot{m}_k \tilde{h}_k + \dot{m}_k \frac{\bar{P}_k}{\bar{\rho}_k} - n_k \cdot q_k^t \right) = - \sum_k \left(\frac{\tilde{v}_k^2 \dot{m}_k}{2} + M_k \cdot \tilde{v}_k \right)$$

or

$$\sum_k (- \dot{m}_k \tilde{h}_k - n_k \cdot q_k^t) = \sum_k \frac{\dot{m}_k}{2} \tilde{v}_k^2 + n_k \cdot v_I P_k - \tilde{v}_k \cdot \tau_k^t \cdot n_k. \quad (54)$$

At low speeds the above interphase balance equation becomes

$$\sum_k (\dot m_k \tilde h_k + n_k \cdot q_k^t) = 0$$

or

$$\sum_k (\langle \dot m_k \tilde h_k \rangle + \langle n_k \cdot q_k^t \rangle) = 0. \tag{55}$$

Momentum equation for crossflow

In order to calculate the crossflow W_{kij} as defined by equation (8b) or phase fluid velocity at the gap, we need to establish a momentum equation applicable at the gap in the direction normal to the gap. The defining equation (8b) implies segment averaging where segment is taken along the gap having a length s_k equal to the intercept between the gap and the phase boundaries. To facilitate this segment averaging we place the coordinate system at the gap as shown in Fig. 1. The application of equations (A.3) and (A.9) to equation (2) yields

$$\begin{aligned}
&\frac{\partial}{\partial t}\int_{s_k}(\rho_k \tilde v_k \mathrm{d}y')_i + \frac{\partial}{\partial x'}\left(\int_{s_k}\bar\rho_k \tilde v_k \tilde v_{kx}\mathrm{d}y'\right)_i \\
&+ \frac{\partial}{\partial z'}\left(\int_{s_k}\bar\rho_k \tilde v_k \tilde v_{kz}\mathrm{d}y'\right)_i \\
&= \frac{\partial}{\partial x'}\left[\int_{s_k}(-I\bar P_k + \tau_k^t)\cdot i\,\mathrm{d}y'\right]_i \\
&+ \frac{\partial}{\partial z'}\left[\int_{s_k}(-I\bar P_k + \tau_k^t)\cdot K\,\mathrm{d}y'\right]_i \\
&+ \sum_{l_I}\left[-\bar\rho_k(\tilde v_k - v_I)\cdot n_k \tilde v_k + (-I\bar P_k + \tau_k^t)\cdot n_k\right] \\
&\times \frac{1}{n_k \cdot n_{kI}} + \sum_{l_w}\tau_k \cdot n_k \frac{1}{n_k \cdot n_{kw}} + \bar\rho_k s_k g_k
\end{aligned} \tag{56}$$

where summation over l_I implies summation over all extremities of the segment s_k at the interface. The x'-component of the above vector equation gives the momentum equation for cross flow. Thus, taking the x'-component of above equation, recognizing the definition (8b) of crossflow, and in addition making use of equation (23) we obtain

$$\begin{aligned}
&\frac{\partial}{\partial t}W_{kij} + \frac{\partial}{\partial x'}\left(\int_{s_k}\bar\rho_k \tilde v_{kx}^2 \mathrm{d}y'\right)_{ij} + \frac{\partial}{\partial z'}\left(\int_{s_k}\bar\rho_k \tilde v_{kx}\tilde v_{kz}\right)_{ij} \\
&= \frac{\partial}{\partial x'}\left[\int_{s_k}\left(-\bar P_k + \tau_{kxx}^t\right)\mathrm{d}y'\right]_{ij} \\
&\quad + \sum_{l_I}\left[M_k \cdot i \frac{1}{n_k \cdot n_{kI}}\right]_{ij} + \sum_{l_w}\left[\tau_{kx}\cdot n_k \frac{1}{n_k \cdot n_{kw}}\right]_{ij}
\end{aligned} \tag{57}$$

where we have assumed the body forces consist of gravity only and hence, $g_x = 0$.

For momentum balance in direction normal to the interface, we have from equation (28b)

$$\left\{\sum_{l_I}\left[-\dot m_1^2\left(\frac{1}{\bar\rho_1} - \frac{1}{\bar\rho_2}\right) - (\bar P_1 - \bar P_2)\right.\right.$$

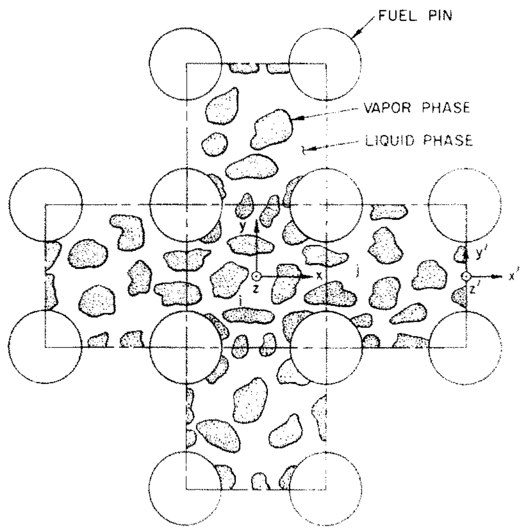

FIG. 1. Two fluid model for a subchannel.

$$\left.\left. + (\tau_{1nn}^t - \tau_{2nn}^t)\right]\frac{1}{n_1 \cdot n_{1I}}\right\}_{ij} = \left\{\sum_{l_I} 2H_{21}\sigma \frac{1}{n_1 \cdot n_{1I}}\right\}_{ij}. \tag{58}$$

For interfacial drag, we utilize equations (34a) and (36) to give

$$\sum_k \sum_{l_I} \tau_{knt}^t \frac{1}{n_1 \cdot n_{1I}} = 0, \tag{59}$$

$$\left\{\sum_k \sum_{l_I}(M_{ke}^t + M_k^n n_{ke})\cdot i \frac{1}{n_1 \cdot n_{1I}} = \sum_k \sum_{l_I} M_{ke}^d \cdot i \frac{1}{n_1 \cdot n_{1I}}\right\}_{ij} = 0. \tag{60}$$

where

$$M_{ke}^t = -(\langle \tau_{knt}^t \rangle_e - \tau_{knt}^t),$$

$$M_k^n = (\langle \bar P_k \rangle_e - \bar P_k) - (\langle \tau_{knn}^t \rangle_e - \tau_{knn}^t), \tag{61a}$$

$$M_{ke}^d = M_{ke}^t + M_{ke}^n n_k, \quad \sum_{l_I} F_k \frac{1}{n_k \cdot n_{kI}} = \langle F_k \rangle_e. \tag{61b}$$

Corresponding to equation (38) we have

$$\begin{aligned}
&\left\{ i \cdot \sum_{l_I} M_k \frac{1}{n_k \cdot n_{kI}} \right. \\
&= i \cdot \left[\sum_{l_I}\left(-\frac{\dot m_k}{\bar\rho_k} - \langle \bar P_k \rangle_e + \langle \tau_{knn}^t \rangle_e\right)\frac{n_{ke}}{n_k \cdot n_{kI}}\right] \\
&+ \sum_{l_I}\langle \tau_{knt}^t \rangle_e \cdot i \frac{1}{n_k \cdot n_{kI}} \\
&\left. - i \cdot \sum_{l_I}\dot m_k v_I \frac{1}{n_k \cdot n_{kI}} + i \cdot \langle M_k^d \rangle_e \right\}_{ij}.
\end{aligned} \tag{62}$$

We may note that

$$-\langle \bar P_k \rangle_e \frac{n_{ke}\cdot i}{n_k \cdot n_{kI}} = \langle \bar P_k \rangle_e \frac{\partial s_k}{\partial x'}.$$

Equation (57), together with equations (58)–(60), and (62) provide the required set of governing equations for calculating crossflow.

SUMMARY AND CONCLUDING REMARKS

We have studied a general formulation of the two-fluid model applicable to a pin bundle. The governing equations for the model are equations (12), (13), (42), (36), (30), (52), (55), (57), (58), and (60). However, we still need several constitutive relationships to close the system of equations. Although a significant progress has been made in obtaining constitutive relationships for drift flux model [25] for flow in pipes, no such claims can be made for two-fluid model. Because of the assumed quasi-two-dimensional nature of flow in pin bundles, a considerably greater number of constitutive relationships are required as compared to the flow in simple geometries, therefore a considerable effort will be required to firmly establish two-fluid model for two-phase flows. We may recall even in a single-phase turbulent flow in a pin bundle, the firm basis for the constitutive relationships for turbulent fluxes at the interchannel boundaries is still lacking. It is for these reasons we have discussed the two-fluid model only in general terms to bring out the various interaction terms and to provide a framework for establishing the constitutive relationship for these terms.

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APPENDIX

The Leibnitz's theorem

For a volume. Let $\mathscr{V}_k$ contained in a channel be cut by two cross-sectional planes $a_k|_z$ and $a_k|_{z+\Delta z}$ located a distance Δz apart. Let v_k be the part of total volume $\mathscr{V}_k$ bounded between two cross-sectional planes and surface area a_I which may include both the interface and the external boundary as shown in Fig. A-1. Let n_k be the unit normal to the surface a_I directed away from phase k. The Leibnitz theorem applicable to volume v_k is given as

$$\frac{\partial}{\partial t}\int_{v_k} F_k \, dv = \int_{v_k} \frac{\partial F_k}{\partial t} dv + \int_{a_I} F_k v_I \cdot n_k \, da \qquad (A.1)$$

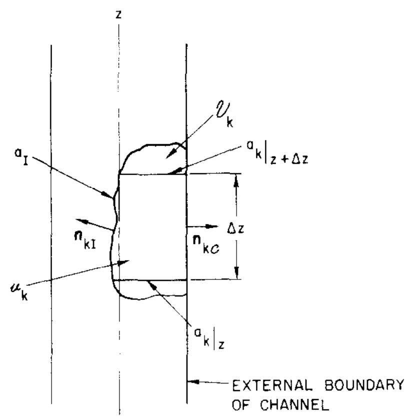

FIG. A.1. Geometry for averaging theorems applicable to a volume v_k.

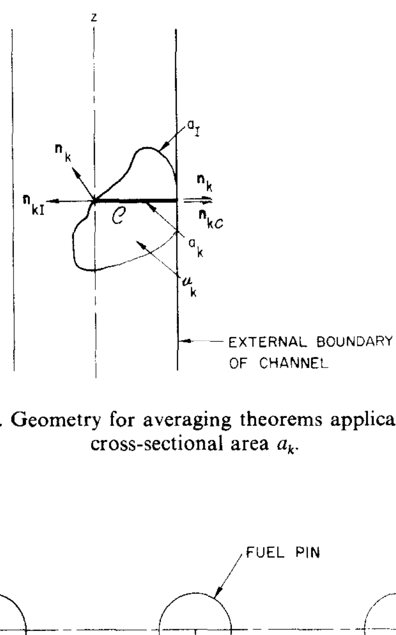

FIG. A.2. Geometry for averaging theorems applicable to a cross-sectional area a_k.

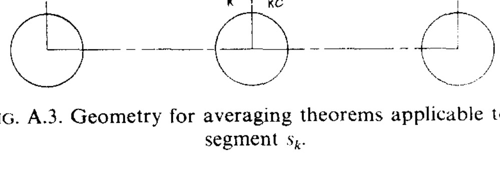

FIG. A.3. Geometry for averaging theorems applicable to a segment s_k.

where $v_I \cdot n_k$ is the displacement velocity of a point on the bounding surface a_I.

For a cross-sectional area. Consider a channel with longitudinal axis along z axis containing a volume v_k of phase k as shown in Fig. A-2. Let a_I be the bounding surface which may both include the interface and the external boundary and n_k be the unit normal directed away from phase k. Let a_k be cross-sectional area cutting volume v_k along the closed curve $\mathscr{C}$ which may include interface as well as the external boundary of the channel. Let n_{kI} be the unit normal to the curve $\mathscr{C}$ in the cross-sectional plane a_k and directed away from phase k. The limiting form of theorem (A.1) with $\Delta z \to 0$ becomes

$$\frac{\partial}{\partial t}\int_{a_k} F_k \, da = \int_{a_k} \frac{\partial F_k}{\partial t} da + \int_{\mathscr{C}} F_k v_I \cdot n_k \frac{d\mathscr{C}}{n_k \cdot n_{kI}}. \tag{A.2}$$

For a segment. Consider a line segment s_k (such as formed by gap between the interconnecting channels at a given axial section at z (see Fig. A-3) cutting the bounding surface a_I of volume v_k at two extremities l_1 and l_2. The limiting form of Leibnitz's theorem for line segment s_k is then given as

$$\frac{\partial}{\partial t}\int_{s_k} F_k \, ds = \int_{s_k} \frac{\partial F_k}{\partial t} ds + \sum_{l_1, l_2} F_k \frac{v_I \cdot n_k}{n_k \cdot n_{kI}}. \tag{A.3}$$

The Gauss theorem

For a volume. The Gauss theorem applicable to volume v_k (see Fig. A-1) is given as

$$\int_{v_k} \nabla \cdot B_k \, dv = \frac{\partial}{\partial z}\int_{v_k} B_{kz} \, dv + \int_{a_I} n_k \cdot B_k \, da \tag{A.4}$$

where B_{kz} is z-component of vector B_k. For a tensor field the Gauss theorem can be stated as

$$\int_{v_k} \nabla \cdot M_k \, dv = \frac{\partial}{\partial z}\int_{v_k} n_z \cdot M_k \, dv + \int_{a_I} n_k \cdot M_k \, da. \tag{A.5}$$

For a cross-sectional area. The Gauss theorem applicable to cross-sectional a_k (see Fig. A-2) becomes

$$\int_{a_k} \nabla \cdot B_k \, da = \frac{\partial}{\partial z}\int_{a_k} B_{kz} \, da + \oint_{\mathscr{C}} n_k \cdot B_k \frac{d\mathscr{C}}{n_k \cdot n_{k\mathscr{C}}}. \tag{A.6}$$

If $B_k = n_z$ the above equation becomes

$$\frac{\partial}{\partial z} a_k = -\oint_{\mathscr{C}} n_k \cdot n_z \frac{d\mathscr{C}}{n_k \cdot n_{k\mathscr{C}}}. \tag{A.7}$$

For tensor fields the Gauss theorem is given as

$$\int_{a_k} \nabla \cdot M_k \, da = \frac{\partial}{\partial z}\int_{a_k} n_z \cdot M_k \, da + \int_{\mathscr{C}} n_k \cdot M_k \frac{d\mathscr{C}}{n_k \cdot n_{k\mathscr{C}}}. \tag{A.8}$$

For a segment. The Gauss theorem for a segment s_k (see Fig. A-3) becomes

$$\int_{s_k} \nabla \cdot B_k \, ds = \frac{\partial}{\partial z}\int_{s_k} B_z \, ds + \frac{\partial}{\partial x}\int_{s_k} B_x \, ds + \sum_{l_1, l_2} n_k \cdot B_k \frac{1}{n_k \cdot n_{k\mathscr{C}}}. \tag{A.9}$$

For a tensor field we have

$$\int_{s_k} \nabla \cdot M_k \, ds = \frac{\partial}{\partial z}\int_{s_k} n_z \cdot M_k \, ds + \frac{\partial}{\partial x} \times \int_{s_k} n_x \cdot M_k \, ds + \sum_{l_1, l_2} n_k \cdot M_k \frac{1}{n_k \cdot n_{k\mathscr{C}}}. \tag{A.10}$$

MODELE A DEUX FLUIDES DE L'ECOULEMENT DIPHASIQUE DANS LA GRAPPE DE BARRE D'UN REACTEUR NUCLEAIRE

Résumé—En considérant un écoulement diphasique comme un champ subdivisé en deux régions turbulentes monophasiques avec des frontières mobiles entre les deux phases, de telle sorte que les bilans différentiels pour l'écoulement turbulent tridimensionnel s'harmonisent pour chaque sous-région et pour l'interface, on établit la moyenne eulérienne dans la section droite de chaque phase dans un canal donné, et la moyenne segmentaire de l'équation de quantité de mouvement transverse aux frontières entre canaux. Pour simplifier les équations obtenues comme résultat de ces opérations, on fait l'hypothèse que le mouvement du fluide dans chaque phase est dominant dans la direction axiale, c'est-à-dire que les composantes transversales de la vitesse sont petites comparées aux composantes axiales. On suppose encore que la variation de la composante axiale de la vitesse dans un canal est plus forte que la variation le long de la direction axiale. On suppose aussi que des arguments semblables peuvent être appliqués à la variation d'enthalpie dans un canal. A partir de ces considérations on obtient deux systèmes d'équations de continuité, de quantité de mouvement et d'énergie pour décrire le mouvement de chaque phase dans la direction axiale. Les termes d'intéraction entre phases qui apparaissent dans ces équations sont gouvernés termes d'intéraction entre phases qui apparaissent dans ces équations sont gouvernés par des conditions de transfert interfacial obtenues à partir des bilans aux interfaces. L'équation de la moyenne segmentaire de la quantité de mouvement transverse pour chaque phase fournit l'équation fondamentale de l'écoulement transversal.

ZWEI-FLUID-MODELL DER ZWEIPHASENSTRÖMUNG IM STABBÜNDEL EINES KERNREAKTORS

Zusammenfassung—Durch die Betrachtung der Zweiphasenströmung als ein Feld, welches in zwei turbulente einphasige Gebiete mit veränderlichen Grenzen unterteilt ist, die die beiden Grundphasen derart trennen, daß die differentiellen Bilanzgleichungen für dreidimensionale turbulente Strömung für jedes Untergebiet und für die Grenzflächen erfüllt sind, können wir die Eulersche Flächenmittelung über die Querschnittsfläche jeder Phase in einem gegebenen Kanal und die abschnittsweise Mittelung der Querimpulsgleichung entlang der Phasenabschnitte an den Grenzen innerhalb des Kanals durchführen. Um die mit Hilfe dieses Verfahrens erhaltenen Hauptgleichungen zu vereinfachen, nehmen wir an, daß die Bewegung des Fluids in jeder Phase in axialer Richtung dominant ist, d. h. die Querkomponenten der Geschwindigkeit sind klein im Vergleich zu den axialen Komponenten.

Wir nehmen weiter an, daß die Veränderung der axialen Komponente der Geschwindigkeit innerhalb eines Kanals viel größer ist als die Veränderung in axialer Richtung. Wir setzen ferner voraus, daß sich die Enthalpieschwankung in einem Kanal ähnlich verhält. Als Ergebnis dieser Überlegungen erhalten wir zwei Gruppen von Kontinuitäts-, Impuls- und Energiegleichungen, welche die Bewegung von jeder Phase in axialer Richtung beschreiben. Die Wechselwirkungs-Terme zwischen den Phasen, die in diesen Gleichungen vorkommen, werden bestimmt durch die Transportbedingungen an der Phasengrenze, die man durch Grenzflächenbilanzen erhält. Die abschnittsweise Mittelung der Querimpulsgleichung für jede Phase führt auf die Hauptgleichung für den Kreuzstrom.

ДВУХЖИДКОСТНАЯ МОДЕЛЬ ДЛЯ ДВУХФАЗНОГО ОБТЕКАНИЯ ПУЧКА ТОНКИХ СТЕРЖНЕЙ ЯДЕРНОГО РЕАКТОРА

Аннотация — На основании рассмотрения двухфазного потока как поля, которое можно разбить на две турбулентные однофазные области с подвижными границами между двумя составляющими фазами таким образом, чтобы дифференциальные уравнения для трехмерного турбулентного потока были справедливы для каждой подобласти и для границы раздела, проведено эйлеровское усреднение по площади поперечного сечения каждой фазы в канале и усреднение уравнения количества движения в поперечном направлении вдоль пересечения фаз на границах между каналами. Для упрощения полученных основных уравнений используется допущение о том, что движение жидкости в каждой фазе доминирует в аксиальном направлении, т. е. поперечные компоненты скорости малы по сравнению с аксиальными. Предполагается также, что изменение аксиальной компоненты скорости поперек канала происходит гораздо быстрее, чем изменение скорости вдоль оси. Высказано предположение о том, что аналогичное упрощение можно использовать для изменения энтальпии в канале. В результате анализа получены две системы уравнений неразрывности, количества движения и энергии, описывающих перемещение каждой фазы в аксиальном направлении. Взаимодействие фаз в этих уравнениях подчиняется условиям переноса на границе раздела, полученным из рассмотрения баланса на границе раздела. Из осредненного уравнения количества движения в поперечном направлении получено основное уравнение для поперечного потока.